Maximizing the Guarded Interior of an Art Gallery

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- The Art Gallery (AG) Theorem
- Variations of the AG Problem
- Visibility Predicates
- Approximability and Inapproximability Results of MVG
- Our Problems
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- A Hierarchy of Approximation Classes
- A Simple Construction
- The Maximum Area Vertex/Edge Guards Problem
- An Approximation Algorithm for Vertex Guards
- The Maximum Treasures Value Vertex Guards Problem
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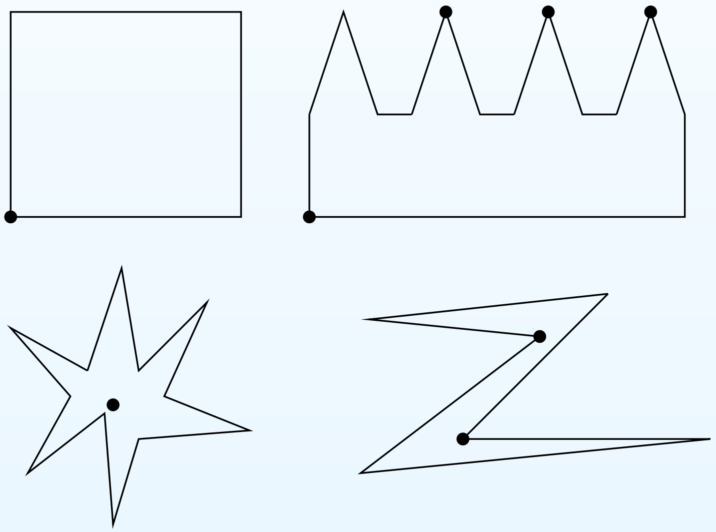
The Art Gallery (AG) Theorem

- AG theorem: $\lfloor \frac{n}{3} \rfloor$ guards always suffice and are sometimes necessary to guard a given polygon with n vertices.



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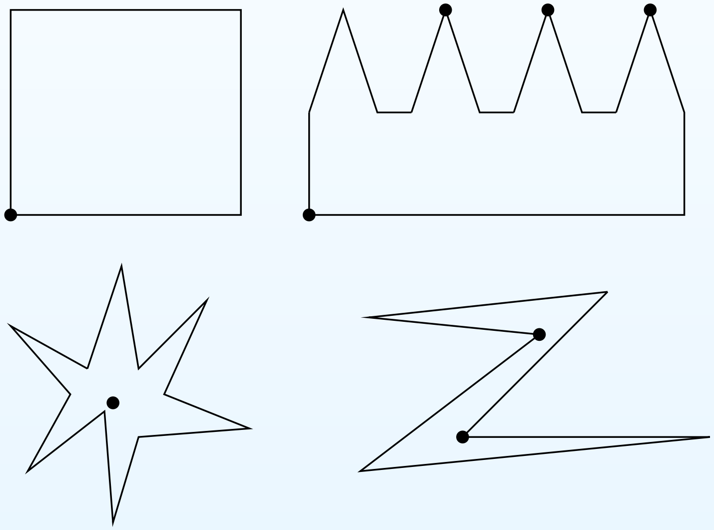

- Minimum Vertex Guards (MVG): place as few as possible guards so that the polygon is guarded (NP-hard in many variations).

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- Vertex Guards
- Point Guards
- Edge Guards
- Point Guards placed inside edges or generaly inside the polygon

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Variations of the AG Problem

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- Point Guards placed inside edges or generaly inside the polygon
- Overseen or watched point sets on the boundary or in the interior

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Variations of the AG Problem

- Vertex Guards
- Point Guards
- Edge Guards
- Point Guards placed inside edges or generaly inside the polygon
- Overseen or watched point sets on the boundary or in the interior
- Polygons with holes

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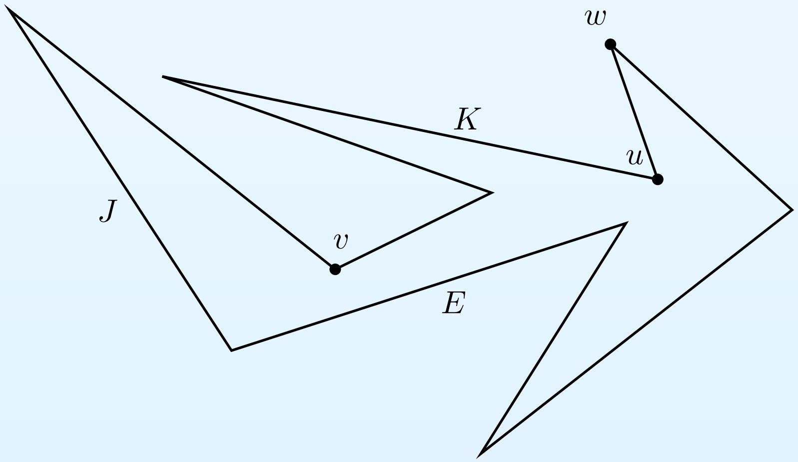

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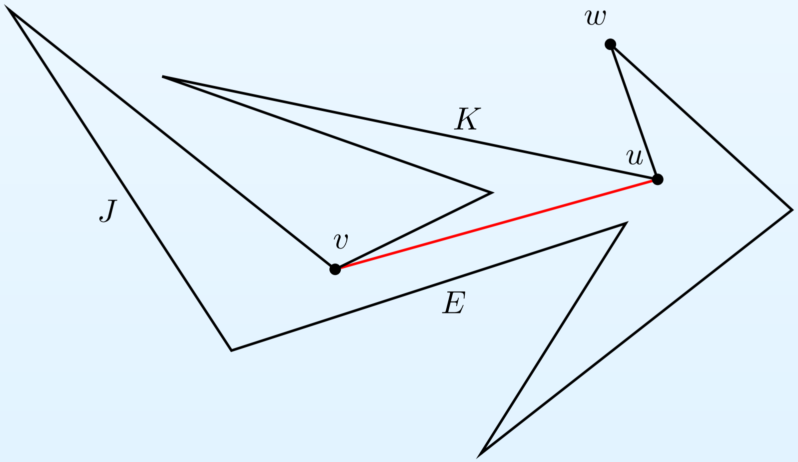

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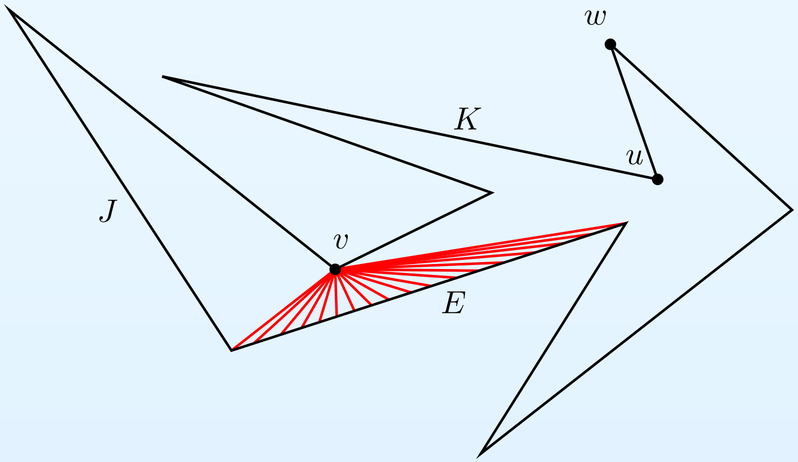

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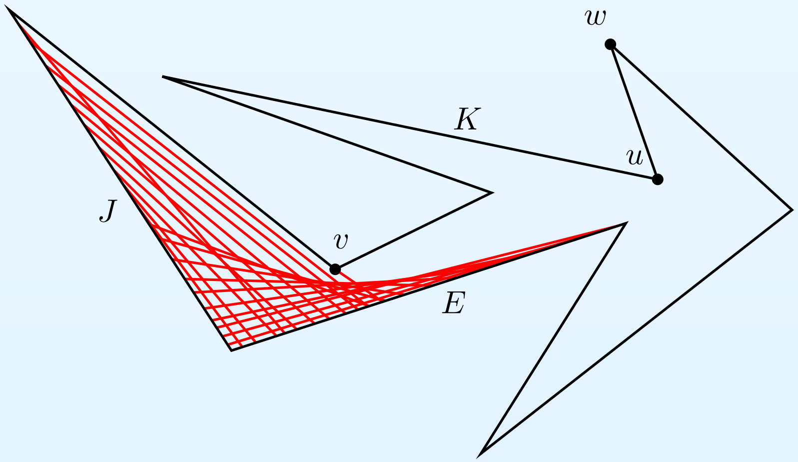

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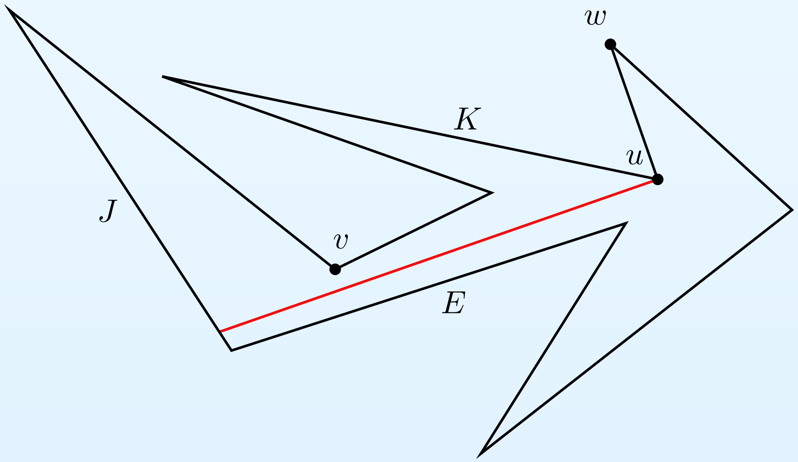

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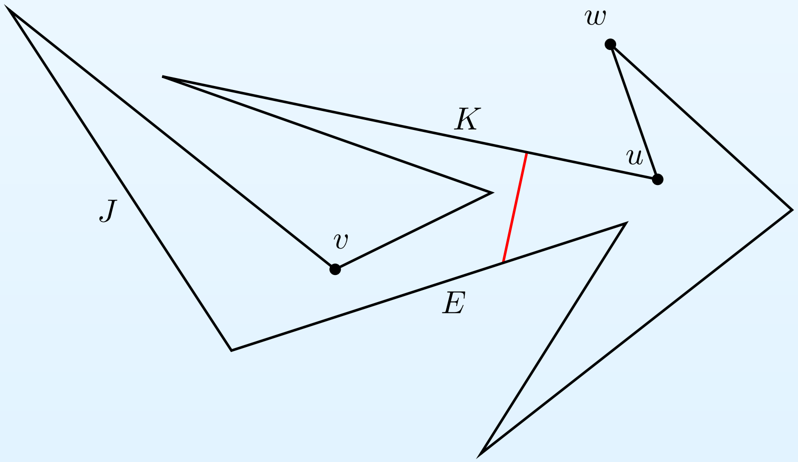

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Approximability and Inapproximability Results of MVG

- The transformation to MINIMUM SET COVER leads to a $\log n$-hard problem.

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- The transformation to MINIMUM SET COVER leads to a $\log n$-hard problem.

- There exist $O(\log n)$ approximation algorithms (Ghosh, 1987):

$$SOL \leq O(\log n) OPT$$

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- MINIMUM VERTEX GUARDS is APX-hard (Eidenbenz, 2000): $\exists \epsilon > 0$: an approximation of $1 + \epsilon$ cannot be guaranteed by any approximation algorithm, unless $P = NP$.

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- The transformation to Minimum Set Cover leads to a $\log n$-hard problem.

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- Minimum Vertex Guards is APX-hard (Eidenbenz, 2000): $\exists \epsilon > 0$: an approximation of $1 + \epsilon$ cannot be guaranteed by any approximation algorithm, unless $P = NP$.

- Problems with holes are $\log n$-hard up to a constant factor.

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- The transformation to MINIMUM SET COVER leads to a $\log n$-hard problem.
- There exist $O(\log n)$ approximation algorithms (Ghosh, 1987):

$$SOL \leq O(\log n) OPT$$

- MINIMUM VERTEX GUARDS is APX-hard (Eidenbenz, 2000): $\exists \epsilon > 0$: an approximation of $1 + \epsilon$ cannot be guaranteed by any approximation algorithm, unless $P = NP$.
- Problems with holes are $\log n$-hard up to a constant factor.
- No constant approximation factors are known.

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Our problems are **maximization** problems:

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Our Problems

Our problems are **maximization** problems:

- Maximum Area Vertex Guards: place k **vertex** guards so that the overseen interior is maximum. Variations: edge guards, polygons with holes.

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Our Problems

Our problems are **maximization** problems:

- MAXIMUM AREA VERTEX GUARDS: place k **vertex** guards so that the overseen interior is maximum. Variations: edge guards, polygons with holes.

- MAXIMUM TREASURES VALUE VERTEX GUARDS: place k **vertex** guards in a “treasury” so that the total value of the overseen or watched treasures is maximum.

The problems are NP-hard: MINIMUM VERTEX GUARDS reduces to our problems.

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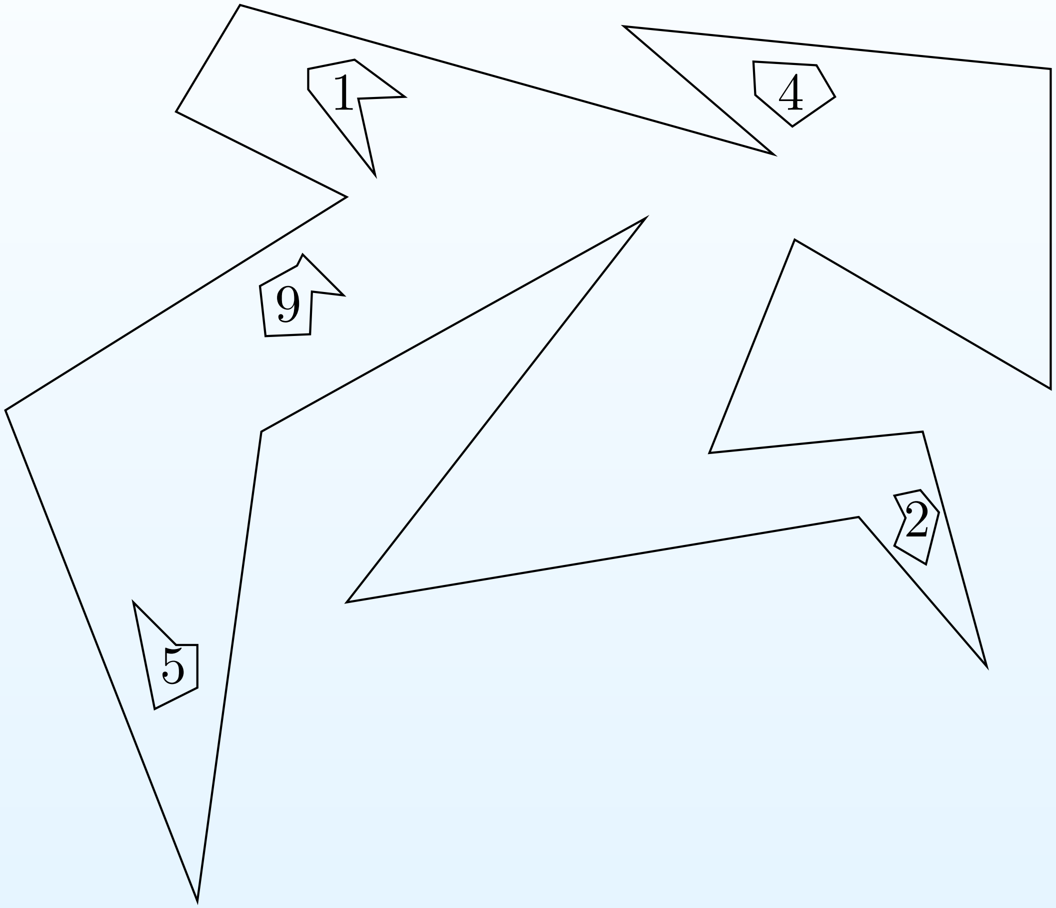

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Previous Work

- MAXIMUM LENGTH VERTEX/EDGE GUARDS: place k **vertex** guards so that the overseen boundary is maximum.

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Previous Work

- MAXIMUM LENGTH VERTEX/EDGE GUARDS: place k **vertex** guards so that the overseen boundary is maximum.

- BUDGETED MAXIMUM LENGTH VERTEX/EDGE GUARDS: place **vertex** guards that do not cost more than a given budget so that the overseen boundary is maximum.

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- MAXIMUM LENGTH VERTEX/EDGE GUARDS: place k **vertex** guards so that the overseen boundary is maximum.
- BUDGETED MAXIMUM LENGTH VERTEX/EDGE GUARDS: place **vertex** guards that do not cost more than a given budget so that the overseen boundary is maximum.
- MAXIMUM VALUE VERTEX/EDGE GUARDS WITH PAINTING PLACEMENT: place paintings and **vertex** guards so that the overseen values is maximum.

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Our Results

- Maximum Length/Area Vertex/Edge Guards (MLAVEG) is NP-hard and can be approximated within a constant (belong to APX class):

$$SOL \geq c\, OPT,\ 1 \geq c > 0$$

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- MLAVEG is APX-hard: $\exists\epsilon > 0$: no polynomial time algorithm can achieve an approximation ratio of $\frac{1}{1+\epsilon}$.

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- MAXIMUM LENGTH/AREA VERTEX/EDGE GUARDS (MLAVEG) is NP-hard and can be approximated within a constant (belong to APX class):

$$SOL \geq c\, OPT,\; 1 \geq c > 0$$

- MLAVEG is APX-hard: $\exists \epsilon > 0$: no polynomial time algorithm can achieve an approximation ratio of $\frac{1}{1+\epsilon}$.
- MLAVEG is APX-complete.

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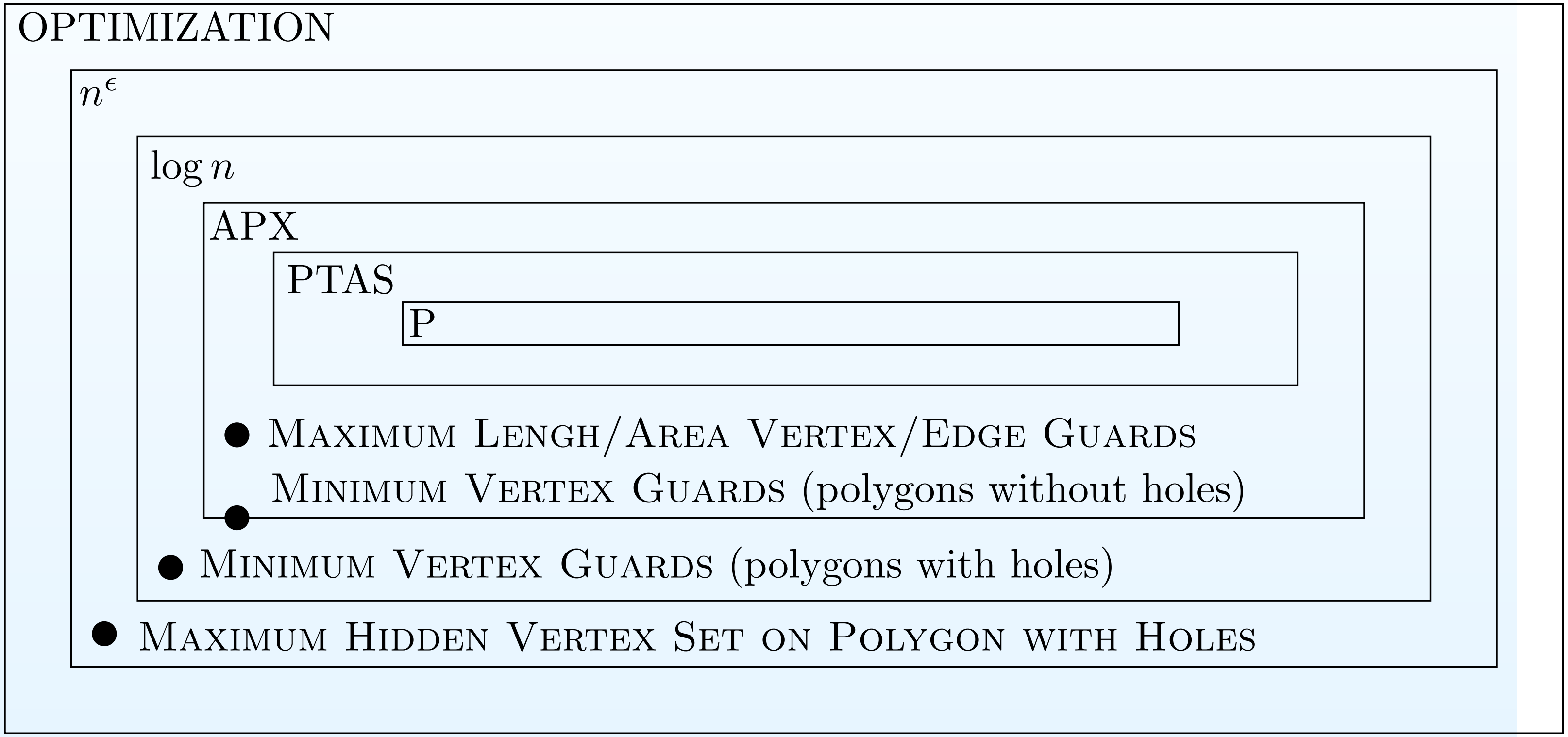

OPTIMIZATION

n^ϵ

$\log n$

APX

PTAS

P

- MAXIMUM LENGH/AREA VERTEX/EDGE GUARDS
- MINIMUM VERTEX GUARDS (polygons without holes)
- MINIMUM VERTEX GUARDS (polygons with holes)
- MAXIMUM HIDDEN VERTEX SET ON POLYGON WITH HOLES

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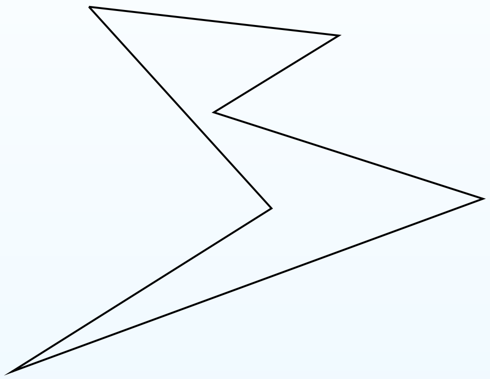

- Start with an arbitrary polygon.

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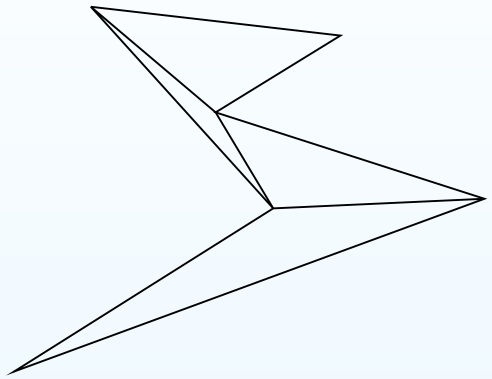

- Start with an arbitrary polygon.
- Find the Visibility Graph.

A Simple Construction

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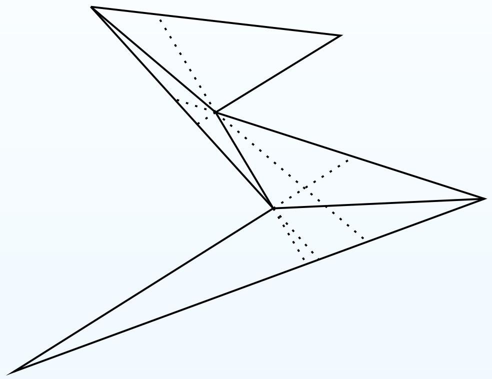

- Start with an arbitrary polygon.
- Find the Visibility Graph.
- Extend the visibility graph's edges, inside and up to the boundary of the polygon.

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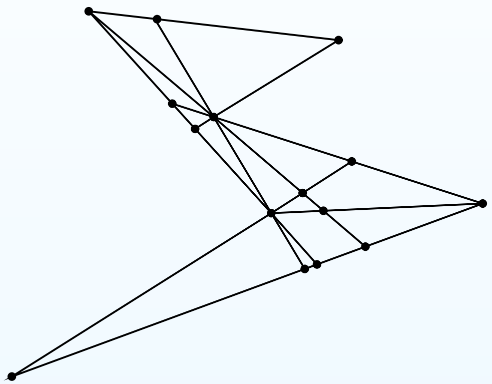

- Start with an arbitrary polygon.
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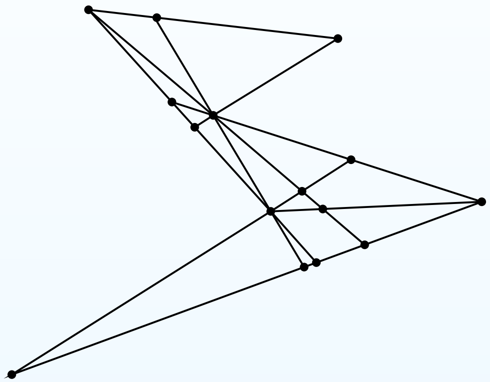

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We discretized the interior with respect to **visibility from the vertices**: an FVS region cannot be **only partly** visible from a vertex or an edge.

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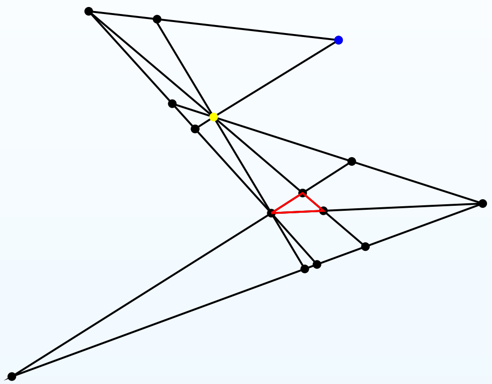

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We discretized the interior with respect to **visibility from the vertices**: an FVS region cannot be **only partly** visible from a vertex or an edge.

Theorem: Any vertex (edge) of P sees a FVS region if and only if watches the FVS region.

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The MAXIMUM AREA VERTEX/EDGE GUARDS Problem

- NP-hardness:
  - Decision Version: Given are a polygon P and integers $k, A > 0$. Can we place at most k vertex or edge guards so that the overseen area is at least A?
  - MINIMUM VERTEX GUARDS (decision version): is the polygon overseen by at most k guards?

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  - MINIMUM VERTEX GUARDS (decision version): is the polygon overseen by at most k guards?
- APX-hardness: “gap-preserving” reduction from MAX-5-OCCURENCE-3-SAT to MAXIMUM AREA VERTEX GUARDS.

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An Approximation Algorithm for Vertex Guards

- $SOL = \emptyset$, calculate FVS regions

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- $\forall v$ calculate $FVS(v)$

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- $SOL = \emptyset$, calculate FVS regions
- $\forall v$ calculate $FVS(v)$
- $\forall v$ without a guard:

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An Approximation Algorithm for Vertex Guards

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- $\forall v$ calculate $FVS(v)$
- $\forall v$ without a guard:
    - select v that maximizes $A(FVS(v) \setminus SOL \cap FVS(v))$

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- $\forall v$ calculate $FVS(v)$
- $\forall v$ without a guard:
  - ○ select v that maximizes $A(FVS(v) \setminus SOL \cap FVS(v))$
  - ○ Update SOL

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- $\forall v$ calculate $FVS(v)$
- $\forall v$ without a guard:
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    - Update SOL
- Return $A(SOL)$

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- $SOL = \emptyset$, calculate FVS regions
- $\forall v$ calculate $FVS(v)$
- $\forall v$ without a guard:
    - select v that maximizes $A(FVS(v) \setminus SOL \cap FVS(v))$
    - Update SOL
- Return $A(SOL)$

Finally $A(SOL) > (1 - \frac{1}{e})\ A(OPT) \approx 0.632\ A(OPT)$.

The MAXIMUM TREASURES VALUE VERTEX GUARDS Problem

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- Decision Version: Given is a treasury P and two integers $k, M > 0$. Can we place at most k vertex guards so that the the overseen value is at least M?

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- NP-hardness: Construct all FVS regions of P, assign value 1 to each region, take as M the number of FVS regions.

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The MAXIMUM TREASURES VALUE VERTEX GUARDS **Problem**

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- APX-hardness and a constant approximation algorithm.

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Open Problems

- How to place guards and given subpolygons in the polygon so that a maximum value is guarded (i.e. this time we need to place also the subpolygons)
- How to place guards in the interior of P for all of our problems.

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